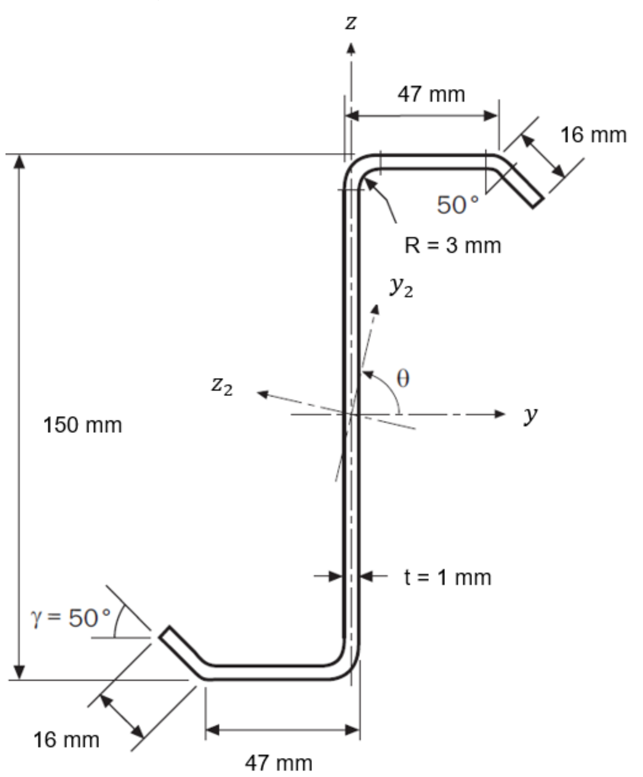
EC3 1-3 2006 CFFD Example 004

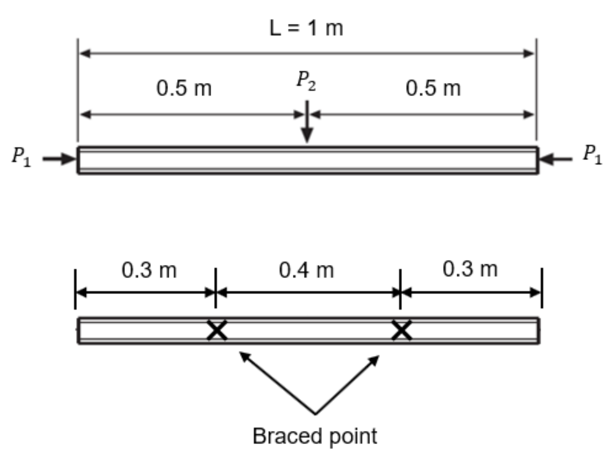
Z-SECTION MEMBER WITH LIPS UNDER COMBINED COMPRESSION, BENDING, AND SHEAR

EXAMPLE DESCRIPTION

Compression, moment, and shear capacities and demand/capacity ratio are calculated for Z section with lips at mid-span as shown below. It is simply supported with a length of 1.0 meters. The member is braced for flexural buckling about z-axis and lateral-torsional buckling at the location of 0.3 meter from each end.

GEOMETRY, PROPERTIES AND LOADING





Dead: $P_1 = 1000\ N, P_2 = 9000\ N$

TECHNICAL FEATURES TESTED

- Axial compressive strength
- Major moment strength
- Shear strength
- Demand/Capacity ratio.

COMPUTER FILE: EC3 1-3 2006 CFFD EX004

Applicable Programs

- SAP2000

RESULTS COMPARISON

Independent results are hand calculated.

CONCLUSION

The results show exact match with independent results.

Benchmarks: SAP2000

Output Parameter	Program	Independent	Percent Difference
Axial - Flexural buckling $N_{b,Rd}\ (N)$	43685	43673	0.03%
Axial – Torsional-Flexural buckling $N_{b,Rd}\ (N)$	44116	44114	0.00%
Axial – Local & Distortional Buckling $N_{c,Rd}\ (N)$	44116	44114	0.00%
Flexure – Lateral-Torsional Buckling $M_{b,Rd}\ (N-mm)$	3110781	3110783	0.00%
Flexure – Local & Distortional Buckling $M_{c,Rd}\ (N-mm)$	3110781	3110783	0.00%
Shear $V_{b,Rd}\ (N)$	7888	7888	0.00%
D/C Ratio	0.954	0.954	0.00%

HAND CALCULATION

Properties:

Material: $E = 210{,}000\ N/mm^2$, $G = 80{,}770\ N/mm^2$, $f_{yb} = 350\ N/mm^2$

Section:

$h = 150\ mm$, $b = 47\ mm$, $t = 1\ mm$, $c = 16\ mm$, $r = 3\ mm$, $\gamma = 50°$

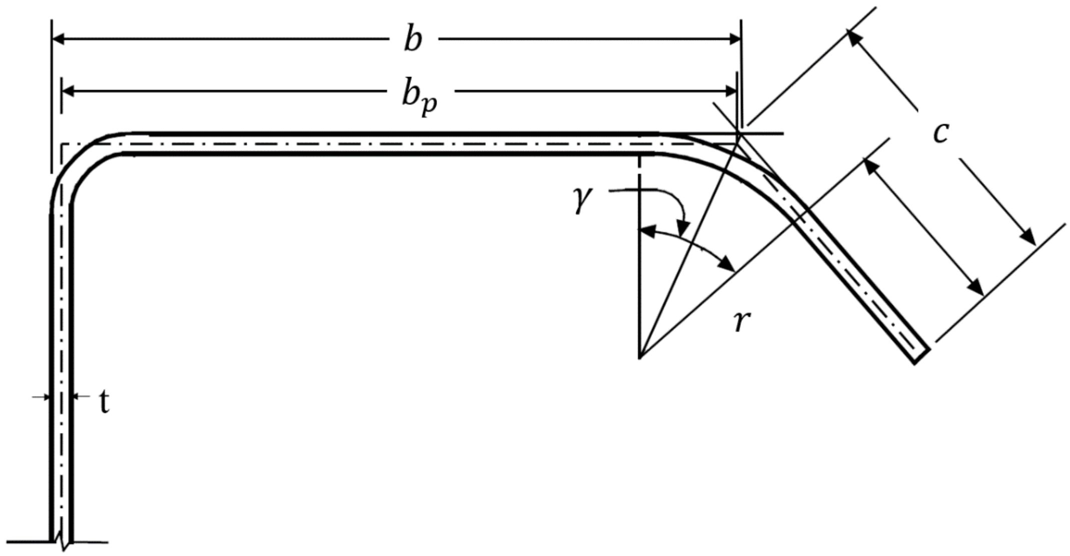


Figure 1: Notional flat width of elements

$\rightarrow h_p = h - t = 150 - 1 = 149\ mm$

$\rightarrow b_p = b - \frac{t}{2} - \frac{t}{2} tan\left(\frac{\gamma}{2}\right) = 47 - \frac{1}{2} - \frac{1}{2} tan\left(\frac{50°}{2}\right) = 46.267\ mm$

$\rightarrow c_p = c - \frac{t}{2} tan\left(\frac{\gamma}{2}\right) = 16 - \frac{1}{2} tan\left(\frac{50°}{2}\right) = 15.767\ mm$

Check for the effect of rounding of the corners:

$\frac{r}{t} = \frac{3}{1} = 3 < 5 \rightarrow OK$

$\frac{r}{b_p} = \frac{3}{46.267} = 0.065 < 0.1 \rightarrow OK$

Therefore, the effect of rounding of the corners can be neglected in calculation of section properties:

$A_g = 273.067\ (mm^2)$

$I_y = 973646.724\ (mm^4)$

$I_z = 149394.063\ (mm^4)$

$i_y = 58.598\ (mm)$

$i_z = 23.39\ (mm)$

$W_{el} = 12585.862\ (mm^3)$

$I_t = 91.022\ (mm^4)$
$I_w = 647151486\ (mm^6)$
$y_0 = z_0 = 0.0\ (mm)$
$\theta = 72.8°$
$I_{y2} = 65793.75\ (mm^4)$
$I_{z2} = 1021247.04\ (mm^4)$
$i_{y2} = 15.522\ (mm)$
$i_{z2} = 61.155\ (mm)$

Member: $K_y = K_z = K_T = 1.0$ for a pinned-pinned condition
$L_y = 3000\ mm$
Member is braced against flexural buckling about z-z axis and lateral-torsional buckling at 300-mm from each end. As a result, the unbraced length of the middle segment in which the mid-span section falls is $L_z = L_{LTB} = 400\ mm$
$k_{yy} = k_{zz} = k_{zy} = k_{yz} = 1.0$

Loadings: Dead: $P_1 = 500\ N, P_2 = 250\ N$
Live: $P_1 = 1000\ N, P_2 = 500\ N$

Required strengths: for the section in the middle

$$N_{Ed} = 1.2D = 1.2 \times 1000 = 1200\ (N)$$
$$M_{Ed} = 1.2D = 1.2 \times \frac{9000 \times 1000}{4} = 2700000\ (N - mm)$$
$$V_{Ed} = 1.2D = 1.2 \times \frac{9000}{2} = 5400\ (N)$$

Member Compression Capacity: the compression capacity is calculated considering the limit states of global buckling, and local and distortional buckling.

1. Local and distortional buckling:
The effective width method is utilized to calculate the nominal axial strength in consideration of local and distortional buckling with the compressive stress of $f_{yb} = 350\ (N/mm^2)$.

Check for the applicability of the method as the following conditions are satisfied:

$$\frac{b}{t} = \frac{47}{1} = 47 < 60 \rightarrow OK$$
$$\frac{c}{t} = \frac{16}{1} = 16 < 50 \rightarrow OK$$
$$\frac{h}{t} = \frac{150}{1} = 150 < 500 \rightarrow OK$$
$$\frac{c}{b} = \frac{16}{47} = 0.34 \rightarrow 0.2 < \frac{c}{b} < 0.6 \rightarrow OK$$

As the section is subjected to uniform compression and both flanges have identical dimension, they are considered as partially stiffened element with a simple lip edge stiffener and have the same effective properties. The calculation below is only shown for the top flange:

$$\psi = 1$$

$$k_\sigma = 4$$

$$\varepsilon = \sqrt{\frac{235}{f_{yb}[N/mm^2]}} = \sqrt{\frac{235}{350}} = 0.8194$$

$$\bar{\lambda}_{p,b} = \frac{b_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{46.267/1}{28.4 \times 0.8194\sqrt{4}} = 0.994 > 0.673$$

$$\rho = \frac{\bar{\lambda}_{p,b} - 0.055(3+\psi)}{\bar{\lambda}_{p,b}^2} = \frac{0.994 - 0.055(3+1)}{0.994^2} = 0.78337 \leq 1.0$$

$$b_{eff} = \rho b_p = 0.78337 \times 46.267 = 36.242\ (mm)$$

$$b_{e1} = b_{e2} = 0.5b_{eff} = 0.5 \times 36.242 = 18.121\ (mm)$$

The lip is considered an outstand (unstiffened) element under uniform compression:

$$\frac{c_p}{b_p} = \frac{15.767}{46.267} = 0.341 < 0.35 \rightarrow k = 0.5$$

$$\bar{\lambda}_{p,c} = \frac{c_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{15.767/1}{28.4 \times 0.8194\sqrt{0.5}} = 0.958 > 0.748$$

$$\rho = \frac{\bar{\lambda}_{p,c} - 0.188}{\bar{\lambda}_{p,b}^2} = \frac{0.958 - 0.188}{0.958^2} = 0.839 \leq 1.0$$

$$c_{eff} = \rho c_p = 0.839 \times 15.767 = 13.227\ (mm)$$

The stiffener consisting of b_{e2} of the flange and c_{eff} of the lip is subjected to distortional buckling (b_{e1} of the flange is not affected by distortional buckling and not included in the iterative procedure below):

1st iteration:

$$b_1 = b_2 = b_p - \frac{t\left[b_{e2}\left(\frac{b_{e2}}{2} + c_{eff}cos\gamma\right) + c_{eff}\frac{c_{eff}}{2}cos\gamma\right]}{t\left(b_{e2} + c_{eff}\right)}$$

$$= 46.267 - \frac{1\left[18.121\left(\frac{18.121}{2} + 13.227cos50°\right) + 13.227\frac{13.227}{2}cos50°\right]}{1(18.121 + 13.227)} = 34.32\ (mm)$$

$$A_{s1} = A_{s2} = t\left(b_{e2} + c_{eff}\right) = 1(18.121 + 13.227) = 31.348\ (mm)$$

$$k_{f1} = \frac{A_{s2}}{A_{s1}} = 1$$

$$K = \frac{Et^3}{4(1-\upsilon^2)}\frac{1}{\left(b_1^2h_p + b_1^3 + 0.5b_1b_2h_pk_f\right)}$$

$$= \frac{210{,}000 \times 1^3}{4(1-0.3^2)} \frac{1}{(34.32^2 \times 149 + 34.32^3 + 0.5 \times 34.32 \times 34.32 \times 149 \times 1)} = 0.19\ (N/mm^2)$$

$$I_s = \frac{t\left(t^2 b_{e2}^2 + 4 b_{e2} c_{eff}^3 - 4 b_{e2} c_{eff}^3 cos^2\gamma + t^2 b_{e2} c_{eff} + c_{eff}^4 - c_{eff}^4 cos^2\gamma\right)}{12\left(b_{e2} + c_{eff}\right)}$$

$$= \frac{1(1^2 \times 18.121^2 + 4 \times 18.121 \times 13.227^3 - 4 \times 18.121 \times 13.227^3 cos^2 50° + 1^2 \times 18.121 \times 13.227 + 13.227^4 - 13.227^4 cos^2 50°)}{12(18.121 + 13.227)}$$

$$= 310.9 (mm^4)$$

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} = \frac{2\sqrt{0.19 \times 210{,}000 \times 310.9}}{31.348} = 224.7\ (N/mm^2)$$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}} = \sqrt{350/224.7} = 1.248 \rightarrow 0.65 < \bar{\lambda}_d < 1.38$$

$$\chi_d = 1.47 - 0.723\bar{\lambda}_d = 1.47 - 0.723 \times 1.248 = 0.5676$$

Since $\chi_d = 0.5676 < 1.0$ → iteration is required.

2nd iteration:

b_{e2} of the flange and c_{eff} of the lip are subjected to reduced stress $\sigma_{com,Ed} = \chi_d f_{yb}/\gamma_{M0}$ such that:

$$\bar{\lambda}_{p,b,red} = \bar{\lambda}_{p,b}\sqrt{\chi_d} = 0.994 \times \sqrt{0.5676} = 0.7489 > 0.673$$

$$\rho = \frac{\bar{\lambda}_{p,b,red} - 0.055(3+\psi)}{\bar{\lambda}_{p,b,red}^2} = \frac{0.7489 - 0.055(3+1)}{0.7489^2} = 0.943 \leq 1.0$$

$$b_{e2} = 0.5 b_{eff} = 0.5\rho b_p = 0.5 \times 0.943 \times 46.267 = 21.815\ (mm)$$

$$\bar{\lambda}_{p,c,red} = \bar{\lambda}_{p,c}\sqrt{\chi_d} = 0.958 \times \sqrt{0.5676} = 0.722 < 0.748 \rightarrow \rho = 1.0$$

$$c_{eff} = \rho c_p = 1.0 \times 15.767 = 15.767\ (mm)$$

$$b_1 = b_2 = b_p - \frac{t\left[b_{e2}\left(\frac{b_{e2}}{2} + c_{eff} cos\gamma\right) + c_{eff}\frac{c_{eff}}{2} cos\gamma\right]}{t\left(b_{e2} + c_{eff}\right)}$$

$$= 46.267 - \frac{1\left[21.815\left(\frac{21.815}{2} + 15.767 cos 50°\right) + 15.767\frac{15.767}{2} cos 50°\right]}{1(21.815 + 15.767)} = 31.93\ (mm)$$

$$A_{s1} = A_{s2} = t\left(b_{e2} + c_{eff}\right) = 1(21.815 + 15.767) = 37.582\ (mm)$$

$$k_{f1} = \frac{A_{s2}}{A_{s1}} = 1$$

$$K = \frac{Et^3}{4(1-\upsilon^2)} \frac{1}{\left(b_1^2 h_p + b_1^3 + 0.5 b_1 b_2 h_p k_f\right)}$$

$$= \frac{210{,}000 \times 1^3}{4(1-0.3^2)} \frac{1}{(31.93^2 \times 149 + 31.93^3 + 0.5 \times 31.93 \times 31.93 \times 149 \times 1)} = 0.22\ (N/mm^2)$$

$$I_s = \frac{t\left(t^2 b_{e2}^2 + 4 b_{e2} c_{eff}^3 - 4 b_{e2} c_{eff}^3 cos^2\gamma + t^2 b_{e2} c_{eff} + c_{eff}^4 - c_{eff}^4 cos^2\gamma\right)}{12\left(b_{e2} + c_{eff}\right)}$$

$$= \frac{1(1^2 \times 21.815^2 + 4 \times 21.815 \times 15.767^3 - 4 \times 21.815 \times 15.767^3 cos^2 50° + 1^2 \times 21.815 \times 15.767 + 15.767^4 - 15.767^4 cos^2 50°)}{12(21.815 + 15.767)}$$

$$= 527.3 (mm^4)$$

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} = \frac{2\sqrt{0.22 \times 210{,}000 \times 527.3}}{37.582} = 263\ (N/mm^2)$$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}} = \sqrt{350/263} = 1.1523 \rightarrow 0.65 < \bar{\lambda}_d < 1.38$$

$$\chi_d = 1.47 - 0.723\bar{\lambda}_d = 1.47 - 0.723 \times 1.1523 = 0.637$$

Since $\chi_d = 0.637 \neq 0.5676$ from previous iteration, more iterations are carried out and the final iteration gives:

$\chi_d = 0.63.$

$b_{e2} = 21.113\ (mm)$

$c_{eff} = 15.581\ (mm)$

$b_{e1} = 18.121\ (mm)$

The web is considered an internal (stiffened) element under uniform compression:

$\psi = 1$

$k_\sigma = 4$

$$\bar{\lambda}_{p,b} = \frac{h_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{149/1}{28.4 \times 0.8194\sqrt{4}} = 3.2 > 0.673$$

$$\rho = \frac{\bar{\lambda}_{p,b} - 0.055(3+\psi)}{\bar{\lambda}_{p,b}^2} = \frac{3.2 - 0.055(3+1)}{3.2^2} = 0.291 \leq 1.0$$

$$h_{eff} = \rho h_p = 0.291 \times 149 = 43.344\ (mm)$$

$$A_{eff} = th_{eff} + 2tb_{e1} + 2\chi_d t(b_{e2} + c_{eff})$$
$$= 1 \times 43.344 + 2 \times 1 \times 18.121 + 2 \times 0.633 \times 1(21.113 + 15.581) = 126.04\ (mm^2)$$

$$A_{eff} = 126.04\ (mm^2) < 266\ (mm^2) = A_g$$

$$\rightarrow N_{c,Rd} = \frac{A_{eff} f_{yb}}{\gamma_{M0}} = \frac{126.04 \times 350}{1.0} = 44114\ (N)$$

Because the section is point symmetric, its effective properties are also point symmetric, resulting in $e_{Ny} = e_{Nz} = 0 \rightarrow \Delta M_{y,Ed} = \Delta M_{z,Ed} = 0$

2. <u>Global buckling</u>: includes flexural buckling and torsional and flexural-torsional buckling
   i. Flexural buckling:
      For member with Z section, the principal axes are considered for global buckling under compression. As the angle between geometric and principal axes $\theta = 72.8°$ counter-clockwise, the effective unbraced lengths about principal y_2- and z_2-axes are taken as those of geometric z- and y-axes, respectively.

$$N_{cr,y2} = \frac{\pi^2 EI_{y2}}{(K_z L_z)^2} = \frac{\pi^2(210{,}000)65793.75}{(1.0 \times 400)^2} = 852282.75\ (N)$$

$$N_{cr,z2} = \frac{\pi^2 EI_{z2}}{\left(K_y L_y\right)^2} = \frac{\pi^2(210{,}000)1021247.04}{(1.0 \times 1000)^2} = 2116653.9\ (N)$$

$$\bar{\lambda}_{y2} = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr,y2}}} = \sqrt{\frac{126.04 \times 350}{852282.75}} = 0.228$$

$$\bar{\lambda}_{z2} = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr,z2}}} = \sqrt{\frac{126.04 \times 350}{2116653.9}} = 0.144$$

For Z section with lips, the buckling curve is b and $\alpha = 0.34$

$$\Phi_{y2} = 0.5\left[1 + \alpha\left(\bar{\lambda}_{y2} - 0.2\right) + \bar{\lambda}_{y2}^2\right] = 0.5[1 + 0.34(0.228 - 0.2) + 0.228^2] = 0.531$$

$$\Phi_{z2} = 0.5\left[1 + \alpha\left(\bar{\lambda}_{z2} - 0.2\right) + \bar{\lambda}_{z2}^2\right] = 0.5[1 + 0.34(0.144 - 0.2) + 0.144^2] = 0.501$$

$$\chi_{y2} = \frac{1}{\Phi_{y2} + \sqrt{\Phi_{y2}^2 - \bar{\lambda}_{y2}^2}} = \frac{1}{0.531 + \sqrt{0.531^2 - 0.228^2}} = 0.99$$

$$\chi_{z2} = \frac{1}{\Phi_{z2} + \sqrt{\Phi_{z2}^2 - \bar{\lambda}_{z2}^2}} = \frac{1}{0.501 + \sqrt{0.501^2 - 0.144^2}} = 1.0$$

$$N_{by2,Rd} = \frac{\chi_{y2} A_{eff} f_{yb}}{\gamma_{M1}} = \frac{0.99 \times 126.04 \times 350}{1.0} = 43673\ (N)$$

$$N_{bz2,Rd} = \frac{\chi_{z2} A_{eff} f_{yb}}{\gamma_{M1}} = \frac{1.0 \times 126.04 \times 350}{1.0} = 44114\ (N)$$

ii. Torsional and flexural-torsional buckling:

$$i_0 = \sqrt{i_{y2}^2 + i_{z2}^2 + y_0^2 + z_0^2} = \sqrt{15.522^2 + 61.155^2 + 0^2 + 0^2} = 63.094\ (mm)$$

$$N_{cr,T} = \frac{1}{i_0^2}\left[GI_t + \frac{\pi^2 EI_w}{L_T^2}\right] = \frac{1}{63.094^2}\left[80{,}770 \times 91.022 + \frac{\pi^2 210{,}000 \times 647151486}{(1.0 \times 400)^2}\right] = 2107704\ (N)$$

$$\rightarrow \bar{\lambda}_T = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr,T}}} = \sqrt{\frac{126.04 \times 350}{2107704}} = 0.145$$

For Z section with lips, the buckling curve for torsional-flexural buckling is b and $\alpha = 0.34$

$$\Phi_T = 0.5\left[1 + \alpha\left(\bar{\lambda}_T - 0.2\right) + \bar{\lambda}_T^2\right] = 0.5[1 + 0.34(0.145 - 0.2) + 0.145^2] = 0.501$$

$$\chi_T = \frac{1}{\Phi_T + \sqrt{\Phi_T^2 - \bar{\lambda}_T^2}} = \frac{1}{0.501 + \sqrt{0.501^2 - 0.145^2}} = 1.0$$

$$N_{bT,Rd} = \frac{\chi_T A_{eff} f_{yb}}{\gamma_{M1}} = \frac{1.0 \times 126.04 \times 350}{1.0} = 44114\ (N)$$

Member Flexural Capacity: the flexural capacity is calculated considering the limit states of lateral-torsional buckling, and local and distortional buckling about geometric axis.

1. Local and distortional buckling:
   The effective width method is utilized to calculate the nominal flexural strength in consideration of local and distortional buckling with the compressive stress in the top flange of $f_{yb} = 350\ (N/mm^2)$. As the section is subjected to positive moment, the top flange is under compression and it is considered a partially stiffened element with a simple lip edge stiffener:

$$\psi = 1$$
$$k_\sigma = 4$$
$$\varepsilon = \sqrt{\frac{235}{f_{yb}[N/mm^2]}} = \sqrt{\frac{235}{350}} = 0.8194$$
$$\bar{\lambda}_{p,b} = \frac{b_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{46.267/1}{28.4 \times 0.8194\sqrt{4}} = 0.994 > 0.673$$
$$\rho = \frac{\bar{\lambda}_{p,b} - 0.055(3+\psi)}{\bar{\lambda}_{p,b}^2} = \frac{0.994 - 0.055(3+1)}{0.994^2} = 0.78337 \le 1.0$$
$$b_{eff} = \rho b_p = 0.78337 \times 46.267 = 36.242\ (mm)$$
$$b_{e1} = b_{e2} = 0.5 b_{eff} = 0.5 \times 36.242 = 18.121\ (mm)$$

The bottom flange is in tension and calculation of its effective width is not carried out.

The lip is considered an outstand (unstiffened) element under uniform compression:

$$\frac{c_p}{b_p} = \frac{15.767}{46.267} = 0.341 < 0.35 \rightarrow k = 0.5$$
$$\bar{\lambda}_{p,c} = \frac{c_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{15.767/1}{28.4 \times 0.8194\sqrt{0.5}} = 0.958 > 0.748$$
$$\rho = \frac{\bar{\lambda}_{p,c} - 0.188}{\bar{\lambda}_{p,b}^2} = \frac{0.958 - 0.188}{0.958^2} = 0.839 \le 1.0$$
$$c_{eff} = \rho c_p = 0.839 \times 15.767 = 13.227\ (mm)$$

The stiffener consisting of b_{e2} of the top flange and c_{eff} of the top lip (Figure 1) is subjected to distortional buckling (b_{e1} of the top flange is not affected by distortional buckling and not included in the iterative procedure below):

1st iteration:

$$b_1 = b_p - \frac{t\left[b_{e2}\left(\frac{b_{e2}}{2} + c_{eff}\cos\gamma\right) + c_{eff}\frac{c_{eff}}{2}\cos\gamma\right]}{t\left(b_{e2} + c_{eff}\right)}$$
$$= 46.267 - \frac{1\left[18.121\left(\frac{18.121}{2} + 13.227\cos 50°\right) + 13.227\frac{13.227}{2}\cos 50°\right]}{1(18.121 + 13.227)} = 34.32\ (mm)$$

$A_{s1} = t(b_{e2} + c_{eff}) = 1(18.121 + 13.227) = 31.348\ (mm)$ (top flange)

$A_{s2} = 0$ as the bottom flange is in tension

$$k_{f1} = \frac{A_{s2}}{A_{s1}} = 0$$

$$K = \frac{Et^3}{4(1-\upsilon^2)} \frac{1}{(b_1^2 h_p + b_1^3 + 0.5 b_1 b_2 h_p k_{f1})}$$

$$= \frac{210{,}000 \times 1^3}{4(1-0.3^2)} \frac{1}{(34.32^2 \times 149 + 34.32^3 + 0)} = 0.2672\ (N/mm^2)$$

$$I_s = \frac{t\left(t^2 b_{e2}^2 + 4 b_{e2} c_{eff}^3 - 4 b_{e2} c_{eff}^3 cos^2\gamma + t^2 b_{e2} c_{eff} + c_{eff}^4 - c_{eff}^4 cos^2\gamma\right)}{12(b_{e2} + c_{eff})}$$

$$= \frac{1(1^2 \times 18.121^2 + 4 \times 18.121 \times 13.227^3 - 4 \times 18.121 \times 13.227^3 cos^2 50° + 1^2 \times 18.121 \times 13.227 + 13.227^4 - 13.227^4 cos^2 50°)}{12(18.121 + 13.227)}$$

$$= 310.9 (mm^4)$$

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} = \frac{2\sqrt{0.2672 \times 210{,}000 \times 310.9}}{31.348} = 266.462\ (N/mm^2)$$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}} = \sqrt{350/266.462} = 1.146 \rightarrow 0.65 < \bar{\lambda}_d < 1.38$$

$$\chi_d = 1.47 - 0.723\bar{\lambda}_d = 1.47 - 0.723 \times 1.146 = 0.6414$$

Since $\chi_d = 0.6414 < 1.0 \rightarrow$ iteration is required.

2nd iteration:

b_{e2} of the flange and c_{eff} of the lip are subjected to reduced stress $\sigma_{com,Ed} = \chi_d f_{yb}/\gamma_{M0}$ such that:

$$\bar{\lambda}_{p,b,red} = \bar{\lambda}_{p,b}\sqrt{\chi_d} = 0.994 \times \sqrt{0.6414} = 0.796 > 0.673$$

$$\rho = \frac{\bar{\lambda}_{p,b,red} - 0.055(3+\psi)}{\bar{\lambda}_{p,b,red}^2} = \frac{0.796 - 0.055(3+1)}{0.796^2} = 0.91 \le 1.0$$

$$b_{e2} = 0.5 b_{eff} = 0.5 \rho b_p = 0.5 \times 0.91 \times 46.267 = 21.05\ (mm)$$

$$\bar{\lambda}_{p,c,red} = \bar{\lambda}_{p,c}\sqrt{\chi_d} = 0.958 \times \sqrt{0.6414} = 0.767 > 0.748$$

$$\rho = \frac{\bar{\lambda}_{p,c,red} - 0.188}{\bar{\lambda}_{p,c,red}^2} = \frac{0.767 - 0.188}{0.767^2} = 0.984 \le 1.0$$

$$c_{eff} = \rho c_p = 0.984 \times 15.767 = 15.513\ (mm)$$

$$b_1 = b_p - \frac{t\left[b_{e2}\left(\frac{b_{e2}}{2} + c_{eff} cos\gamma\right) + c_{eff}\frac{c_{eff}}{2} cos\gamma\right]}{t(b_{e2} + c_{eff})}$$

$$= 46.267 - \frac{1\left[21.05\left(\frac{21.05}{2} + 15.513 cos50°\right) + 15.513\frac{15.513}{2} cos50°\right]}{1(21.05 + 15.513)} = 32.36\ (mm)$$

$A_{s1} = t(b_{e2} + c_{eff}) = 1(21.05 + 15.513) = 36.56\ (mm)$ (top flange)

$A_{s2} = 0$ as the bottom flange is in tension

$$k_{f1} = \frac{A_{s2}}{A_{s1}} = 0$$

$$K = \frac{Et^3}{4(1-\upsilon^2)} \frac{1}{\left(b_1^2 h_p + b_1^3 + 0.5 b_1 b_2 h_p k_{f1}\right)}$$

$$= \frac{210{,}000 \times 1^3}{4(1-0.3^2)} \frac{1}{(32.36^2 \times 149 + 32.36^3 + 0)} = 0.3038\ (N/mm^2)$$

$$I_s = \frac{t\left(t^2 b_{e2}^2 + 4 b_{e2} c_{eff}^3 - 4 b_{e2} c_{eff}^3 cos^2\gamma + t^2 b_{e2} c_{eff} + c_{eff}^4 - c_{eff}^4 cos^2\gamma\right)}{12\left(b_{e2} + c_{eff}\right)}$$

$$= \frac{1(1^2 \times 21.05^2 + 4 \times 21.05 \times 15.513^3 - 4 \times 21.05 \times 15.513^3 cos^2 50° + 1^2 \times 21.05 \times 15.513 + 15.513^4 - 15.513^4 cos^2 50°)}{12(21.05 + 15.513)}$$

$$= 499.5 (mm^4)$$

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} = \frac{2\sqrt{0.3038 \times 210{,}000 \times 499.5}}{36.56} = 308.9\ (N/mm^2)$$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}} = \sqrt{350/308.9} = 1.064 \rightarrow 0.65 < \bar{\lambda}_d < 1.38$$

$$\chi_d = 1.47 - 0.723\bar{\lambda}_d = 1.47 - 0.723 \times 1.064 = 0.7$$

Since $\chi_d = 0.7 \neq 0.6414$ from previous iteration, more iterations are carried out and the final iteration gives:

$$\chi_d = 0.691.$$
$$b_{e2} = 20.543\ (mm)$$
$$c_{eff} = 15.124\ (mm)$$
$$b_{e1} = 18.121\ (mm)$$

The neutral axis of the section with effective top flange and lip measured from the centerline of the top flange is:

$$\bar{y} = \frac{\sum_i A_i y_i}{A} = \frac{tb_{e1} \times 0 + \chi_d tb_{e2} \times 0 + \chi_d tc_{eff} \times \frac{c_{eff}}{2} cos\gamma + th_p \frac{h_p}{2} + tb_p h_p + tc_p\left(h_p - \frac{c_p}{2} cos\gamma\right)}{tb_{e1} + \chi_d tb_{e2} + \chi_d tc_{eff} + th_p + tb_p + tc_p}$$

$$= \frac{0.691 \times 1 \times 15.124 \times \frac{15.124}{2} cos 50° + 1 \times 149 \frac{149}{2} + 1 \times 46.267 \times 149 + 1 \times 15.767\left(149 - \frac{15.767}{2} cos 50°\right)}{1 \times 18.121 + 0.691 \times 1 \times 20.543 + 0.691 \times 1 \times 15.124 + 1 \times 149 + 1 \times 46.267 + 1 \times 15.767}$$

$$= \frac{20314.466}{253.8} = 80.02\ (mm)$$

The web is considered as internal (stiffened) element under stress gradient:

$$\sigma_1 = f_{yb} = 350\ (N/mm^2)$$

$$\sigma_2 = -f_{yb} \frac{149 - 80.02}{80.02} = -301.7\ (N/mm^2)$$

$$\psi = \frac{\sigma_2}{\sigma_1} = -\frac{301.7}{350} = -0.862$$

$$k_\sigma = 7.81 - 6.29\psi + 9.78\psi^2 = 7.81 - 6.29 \times (-0.862) + 9.78 \times (-0.862)^2 = 20.5$$

$$\bar{\lambda}_{p,b} = \frac{h_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{149/1}{28.4 \times 0.8194\sqrt{20.5}} = 1.414 > 0.673$$
$$\rho = \frac{\bar{\lambda}_{p,b} - 0.055(3+\psi)}{\bar{\lambda}_{p,b}^2} = \frac{1.414 - 0.055(3 - 0.862)}{1.414^2} = 0.6484 \le 1.0$$
$$h_c = \frac{h_p}{(1-\psi)} = \frac{149}{(1+0.862)} = 80.02\ (mm)$$
$$h_t = h_p - h_c = 149 - 80.02 = 68.98\ (mm)$$
$$h_{eff} = \rho\frac{h_p}{(1-\psi)} = 0.65 \times \frac{149}{(1+0.862)} = 51.88\ (mm)$$
$$h_{e1} = 0.4h_{eff} = 0.4 \times 51.88 = 20.752\ (mm)$$
$$h_{e2} = 0.6h_{eff} = 0.6 \times 51.88 = 31.128\ (mm)$$

The neutral axis of the section with effective top flange and lip measured from the centerline of the top flange is:

$$\bar{y} = \frac{\sum_i A_i y_i}{A} = 85.6565\ (mm)$$
$$W_{eff,c} = 8887.95\ (mm^3)$$
$$W_{eff,t} = 12018.77\ (mm^3)$$
$$M_{c,Rd,com} = \frac{W_{eff,c} f_{yb}}{\gamma_{M0}} = \frac{8887.95 \times 350}{1.0} = 3110782.5\ (N-mm)$$
$$M_{c,Rd,ten} = \frac{W_{eff,t} f_{yb}}{\gamma_{M0}} = \frac{12018.77 \times 350}{1.0} = 4206569\ (N-mm)$$

The cross-section consisting of effective flanges and fully effective web under positive moment is shown in Figure 2. The plastic neutral axis y_{pl} computed by equating the total compression and tension forces as follows:

$$\left(ty_{pl} + tb_{e1} + \chi_d tb_{e2} + \chi_d tc_{eff}\right)f_{yb} = \left[t\left(h_p - y_{pl}\right) + tb_p + tc_p\right]f_{yb}$$
$$\rightarrow \left(y_{pl} + b_{e1} + \chi_d b_{e2} + \chi_d c_{eff}\right) = \left[\left(h_p - y_{pl}\right) + b_p + c_p\right]$$
$$\rightarrow \left(y_{pl} + 18.121 + 0.691 \times 20.543 + 0.691 \times 15.124\right)$$
$$= \left[\left(149 - y_{pl}\right) + 46.267 + 15.767\right] \rightarrow\ y_{pl} = 84.134\ mm$$

$$W_{pl,f} = \left[y_{pl}(tb_{e1} + \chi_d tb_{e2}) + \chi_d tc_{eff}\left(y_{pl} - \frac{c_{eff}}{2}sin50°\right)\right] + \left[tb_p\left(h_p - y_{pl}\right) + tc_p\left(h_p - y_{pl} - \frac{c_p}{2}sin50°\right)\right]$$
$$= \left[84.134(1 \times 18.121 + 0.691 \times 1 \times 20.543) + 0.691 \times 1 \times 15.124\left(84.134 - \frac{15.124}{2}sin50°\right)\right]$$
$$+ \left[1 \times 46.267(149 - 84.134) + 1 \times 15.767\left(149 - 84.134 - \frac{15.767}{2}sin50°\right)\right]$$
$$= 3537.6 + 3928.7 = 7466.5\ (mm^3)$$
$$W_{pl,wf} = W_{pl,f} + y_{pl}t\frac{y_{pl}}{2} + \left(h_p - y_{pl}\right)t\frac{\left(h_p - y_{pl}\right)}{2}$$

$$= 7466.5 + 84.134 \times 1 \times \frac{84.134}{2} + (149 - 84.134) \times 1 \times \frac{(149 - 84.134)}{2}$$
$$= 13109\ (mm^3)$$

$$M_{f,Rd} = \frac{W_{pl,f} f_{yb}}{\gamma_{M0}} = \frac{7466.5 \times 350}{1.0} = 2613275\ (N - mm)$$

$$M_{wf,Rd} = \frac{W_{pl,wf} f_{yb}}{\gamma_{M0}} = \frac{13109 \times 350}{1.0} = 4588150\ (N - mm)$$

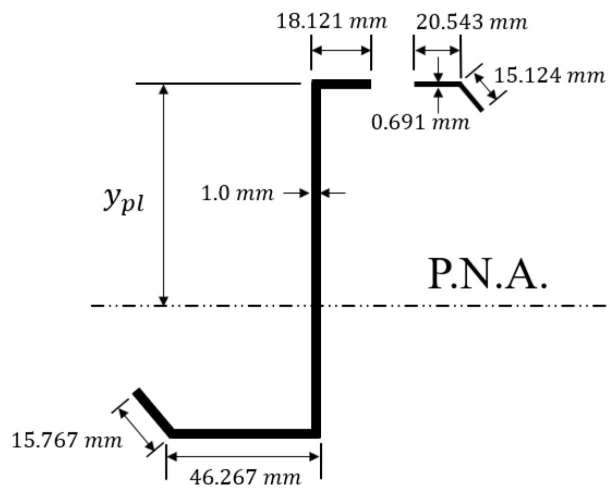


Figure 2: Section with effective flanges and fully effective web under bending

2. Lateral-torsional buckling:
   Between the lateral restraint points that are 300 mm and 700 mm from the left end of the beam, there is one transverse concentrated load $P_2 = 9000\ N$. Even though it is not pinned support condition at these braced points, the coefficients C_1, C_2, and C_3 are taken as those for the simply supported condition to be conservative:
   $C_1 = 1.365$, $C_2 = 0.553$, $C_3 = 1.73$
   $k_w = 1.0$ and $K_{LTB} = 1.0$
   $z_a = 74.5\ (mm)$ as the load is applied on the top flange
   $z_g = z_a - z_s = 74.5 - 0 = 74.5\ (mm)$
   $z_j = 0.0\ (mm)$
   $L_{cr} = 400\ (mm)$
   $I_t = 91.022\ (mm^4)$
   $I_z = 149394.063\ (mm^4)$
   $I_w = 647151486\ (mm^6)$

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{L_{cr}^2}\left\{\left[\left(\frac{K_{LTB}}{k_w}\right)\frac{I_w}{I_z} + \frac{L_{cr}^2 G I_T}{\pi^2 E I_z} + \left(C_2 z_g - C_3 z_j\right)^2\right]^{0.5} - \left(C_2 z_g - C_3 z_j\right)\right\}$$

$$= 96348255\ (N - mm)$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{eff,y} f_{yb}}{M_{cr}}} = \sqrt{\frac{8887.95 \times 350}{96348255}} = 0.18$$

The applicable buckling curve is b and $\alpha_{LT} = 0.34$

$$\Phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right] = 0.5[1 + 0.34(0.18 - 0.2) + 0.18^2] = 0.513$$

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} = \frac{1}{0.513 + \sqrt{0.513^2 - 0.18^2}} = 1.0$$

$$M_{b,Rd} = \chi_{LT} W_{eff,y} \frac{f_{yb}}{\gamma_{M1}} = 1.0 \times 8887.95 \frac{350}{1.0} = 3110782.5\ (N - mm)$$

Member Shear Capacity:

$$\bar{\lambda}_w = 0.346 \frac{s_w}{t}\sqrt{\frac{f_{yb}}{E}} = 0.346 \frac{149}{1}\sqrt{\frac{350}{210000}} = 2.1047$$

$$f_{bv} = \frac{0.67 f_{yb}}{\bar{\lambda}_w^2} = \frac{0.67 \times 350}{2.1047^2} = 52.937\ (N/mm^2)$$

$$V_{b,Rd} = \frac{h_w t f_{bv}}{\gamma_{M0}} = \frac{149 \times 1 \times 52.937}{1.0} = 7888\ (N)$$

Combined D/C ratio:

Eq. 6.25 in EC3 1-3:

$$\frac{D}{C} = \frac{N_{Ed}}{N_{c,Rd}} + \frac{M_{y,Ed} + \Delta M_{y,Ed}}{M_{cy.Rd,com}} + \frac{M_{z,Ed} + \Delta M_{z,Ed}}{M_{cz.Rd,com}} = \frac{1200}{44114} + \frac{2700000 + 0}{3110782.5} + 0 = 0.895$$

Eq. 6.26 in EC3 1-3:

$$\frac{D}{C} = \frac{M_{y,Ed} + \Delta M_{y,Ed}}{M_{cy.Rd,ten}} + \frac{M_{z,Ed} + \Delta M_{z,Ed}}{M_{cz.Rd,ten}} - \frac{N_{Ed}}{N_{c,Rd}} = \frac{2700000 + 0}{4206569} + 0 - \frac{1200}{44114} = 0.615$$

Eq. 6.27 in EC3 1-3:

$$\frac{D}{C} = \frac{N_{Ed}}{N_{Rd}} + \frac{M_{y,Ed}}{M_{y.Rd}} + \left(1 - \frac{M_{f,Rd}}{M_{wf.Rd}}\right)\left(\frac{2V_{y,Ed}}{V_{wy.Rd}} - 1\right)^2 + \frac{M_{z,Ed}}{M_{z.Rd}}$$

$$= \frac{1200}{44114} + \frac{2700000}{3110782.5} + \left(1 - \frac{2613275}{4588150}\right)\left(\frac{2 \times 5400}{7888} - 1\right)^2 + 0 = 0.954$$

Eq. 6.61 in EC3 1-1:

$$\frac{D}{C}=\frac{N_{Ed}}{\frac{\chi_{y2}N_{Rk}}{\gamma_{M1}}}+k_{yy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\frac{\chi_{LT}M_{y,Rk}}{\gamma_{M1}}}+k_{yz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}}$$

$$=\frac{1200}{43673}+1.0\frac{2700000+0}{3110782.5}+0=0.895$$

Eq. 6.62 in EC3 1-1:

$$\frac{D}{C}=\frac{N_{Ed}}{\frac{\chi_{z2}N_{Rk}}{\gamma_{M1}}}+k_{zy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\frac{\chi_{LT}M_{y,Rk}}{\gamma_{M1}}}+k_{zz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}}$$

$$=\frac{1200}{44114}+1.0\frac{2700000+0}{3110782.5}+0=0.895$$

Eq. 6.36 in EC3 1-3:

$$\frac{D}{C}=\left(\frac{N_{Ed}}{N_{b,Rd}}\right)^{0.8}+\left(\frac{M_{y,Ed}+\Delta M_{y,Ed}}{M_{by,Rd}}\right)^{0.8}+\left(\frac{M_{z,Ed}+\Delta M_{z,Ed}}{M_{bz,Rd}}\right)^{0.8}$$

$$=\left(\frac{1200}{43673}\right)^{0.8}+\left(\frac{2700000+0}{3110782.5}\right)^{0.8}+(0)^{0.8}=0.949$$

Equation 6.27 provides the largest D/C ratio D/C = 0.954 that governs the design.